Percolation Effects on Electrical Resistivity and Electron Mobility

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Percolation, defined as the study of transport across a porous material, can be used to determine transport quantities of a material such as electrical and thermal conductivity. Observing the way electrons flow across a porous conductive sheet is a common way that percolation studies are performed, and has many practical applications in electronics. This study determines how elliptical pores cut into a conductive sheet affect the percolation threshold; the remaining area of the conductive sheet when the current across it first becomes zero. Past research has been performed on this topic which yielded theoretical results of the percolation thresholds, and this study aims to verify those results experimentally. This study shows that as the aspect ratio of an ellipse approaches zero, the percolation threshold approaches one. This report also establishes a novel experimental method of studying percolating networks.

Introduction

Percolation is the study of random network systems and the nature of the connectivity between these networks. The study of percolation has many practical applications that range over a wide variety of disciplines. One such discipline is the study of mass movement across a porous medium. In geology, percolation is used to study how water and other nutrients diffuse through soil, while biologists are interested in how medical drugs can diffuse through a cell membrane.1,2 It is desired to see how electron mobility is affected across a conductive sheet that is percolating. Studying electron mobility across a percolating sheet has practical applications including examining the electrical and thermal conductivity of an electronic device, particularly semiconductors.3

For the sake of this research, only systems that are percolating were examined. A system is determined to be percolating when it reaches a percolation threshold. Imagine that there is a sheet of conductive material over which electrons can flow, and holes are being cut into the conductive sheet at random locations. As pores are being cut, pathways for the electrons to flow across are being eliminated, and the current decreases as the area of the sheet is simultaneously decreasing as well. The maximum remaining area of the sheet at which no current can flow from one end to the other end is defined as the percolation threshold.4

Elliptical holes are of interest to this study, particularly how the aspect ratio of an ellipse affects the percolation threshold. The aspect ratio is defined as the length of the minor axis $b$ over the length of the major axis $a$. An aspect ratio ranges between 1 inclusive and zero exclusive. An aspect ratio of 1 is a perfect circle, while as the aspect ratio approaches zero the ellipse becomes a straight line.5 Figure 1 shows examples of three ellipses with different aspect ratios.

Methods

Creating Random Systems of Networks

In order to perform percolation studies, it is necessary to create a completely random system of networks to be studied. This was done using Matlab software which generated shapes randomly within a square area until the percolation threshold is met. The generated shapes block off pathways across the square sheet until there is no connecting pathway that spans from left and right across the sheet. The Matlab code also allows for the different parameters of interest to be controlled.

These random systems of networks that Matlab created are written to a script file which AutoCAD uses to draw the shapes. An example of a drawing is shown using an elliptical pore in Figure 2. There are two sections that are created every time this program runs; the left section gives the percolating sheet, while the right section gives the equivalent area remaining in the porous sheet. As each pore is cut into the sheet, the pore takes away a certain amount of area from the sheet. When the area in the percolation sheet is reduced by a threshold value of 0.1% or more from the porous cuts, the area on the equivalent area sheet was decreased proportionally. This allowed for the area of the percolating sheet to be determined at any point during the cutting. This is useful as

![FIG. 1: Three ellipses with different aspect ratios. From left to right, the aspect ratios are 0.0125, 0.150, and 0.700. Notice that as the aspect ratio approaches zero, the ellipse becomes a straight line, and aspect ratios closer to one become more circular.](image)
FIG. 2: Example of a random system of elliptical pores created by Matlab and drawn by AutoCAD. The left side gives the holes to be cut, while the right side displays the area reduction occurring in the left side.

the current may reach zero before all of the ellipses are cut into the sheet.

Once the shapes were drawn in AutoCAD, they could be outputted to a laser, which would accurately cut the shapes out of aluminum coated Mylar film. Connecting the aluminum coated Mylar film to electrodes leading to a data acquisition unit allows the resistance of the film to be recorded against time as pores are being cut into it. Each time a pore is cut into the left side, the area of the left sheet the pore removed is then removed from the right sheet. This allowed the resistance of the film to also be determined as a function of the equivalent area remaining.

Comparing theoretical and experimental results

Several studies have previously determined theoretical percolation thresholds using different pore parameters. Xia and Thorpe found the theoretical percolation threshold for different aspect ratios of ellipses using computer simulations. As the results they obtained were only theoretical, this study obtained experimental results to compare against their theoretical results. Their theoretically obtained data is shown along with the experimentally obtained data in Table 1 in the results section below.

In performing these experiments to determine the effect of an aspect ratio, all ellipses within one trial have the same aspect ratio. In order to ensure that no factors other than the aspect ratio were affecting the percolation threshold, a constant number of 1,000 ellipses were cut for each trial with different aspect ratios.

Results

Figure 3 shows an example of data that was acquired from a single trial performed. The current across the square sheet was measured as a function of time as the cuts were being made. Knowing the time it takes to perform the experiment, and the total number of cuts, this can then be plotted with current as a function of pores cut. The dashed line of Figure 3 is the current through the porous side of the aluminum sheet, while the solid line is the current through the right side where the equivalent area is being displayed.

After the data had been acquired, the percolation threshold was determined by measuring the resistance change of the equivalent area section at the point when the percolation sections current becomes zero and resistance becomes infinite. As mentioned above, the experimental percolation threshold of ellipses with different aspect ratios was measured and compared to theoretical ones. A comparison graph of these two threshold values is shown below in Figure 4.

Discussion

It is noticed from Table 1 that the experimental percolation thresholds are in good agreement with the theoretical thresholds. Some of the experimental values came to be approximately the exact theoretical value predicted, such as the aspect ratios of 0.1 and 0.1500. It is also noticed though that some of the experimental values were far off from theoretical, not taking error into account. The highest of these is the aspect ratio of 0.4000 which had an experimental threshold off from the theoretical by 10% area. While this seems like a big difference, some error is expected due to the random nature at which the pores are being cut. The rest of this error can be ex-
TABLE I: Theoretical percolation threshold compared to the experimental percolation threshold of ellipses with different aspect ratios. The theoretical percolation thresholds are from computer simulations done by Xia and Thorpe. A wide range of aspect ratios was used to give a better understanding of how it affects a threshold.

<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Theoretical Threshold</th>
<th>Experimental Threshold</th>
<th>Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.33</td>
<td>0.351</td>
<td>0.513</td>
</tr>
<tr>
<td>0.7000</td>
<td>0.34</td>
<td>0.392</td>
<td>0.622</td>
</tr>
<tr>
<td>0.6000</td>
<td>0.35</td>
<td>0.414</td>
<td>0.684</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.41</td>
<td>0.512</td>
<td>0.897</td>
</tr>
<tr>
<td>0.2500</td>
<td>0.50</td>
<td>0.588</td>
<td>1.282</td>
</tr>
<tr>
<td>0.1500</td>
<td>0.62</td>
<td>0.622</td>
<td>1.966</td>
</tr>
<tr>
<td>0.1000</td>
<td>0.70</td>
<td>0.723</td>
<td>2.822</td>
</tr>
<tr>
<td>0.0500</td>
<td>0.83</td>
<td>0.856</td>
<td>5.387</td>
</tr>
<tr>
<td>0.0250</td>
<td>0.91</td>
<td>0.912</td>
<td>10.518</td>
</tr>
<tr>
<td>0.0125</td>
<td>0.95</td>
<td>0.923</td>
<td>20.780</td>
</tr>
</tbody>
</table>

The experiments did verify that as the aspect ratio decreases, the percolation threshold will increase. Both the theoretical and experimental results agreed that each smaller step of an aspect ratio brought about a larger value in percolation threshold. The reason why this occurs is because as the aspect ratio approaches zero, the ellipse approaches a straight line, as shown in Figure 1. Cutting a straight line still blocks electrons from passing, thus lowering the current, but a line has very small area compared to a circle.

It is also worth noting that in performing these tests, the kerf of the laser was approximated as 50 µm. It was desired to see if this approximation was accurate, as the Matlab code uses the kerf to calculate the remaining area of the conductive sheet. It was found that the actual kerf of the laser was 180 µm. This difference resulted in a certain amount of error for each trial performed, where the amount of error depended on the aspect ratio of the ellipse. This is because as the ellipse becomes more of a straight line, the kerf affects the area cut more than a more circular ellipse. An aspect ratio of 0.2500, for example, led to 1.282 % of error, leading to an uncertainty in the percolation thresholds measured of 0.0128. This amount is determined to be negligible. However, an aspect ratio of 0.0125 had 20.780 % error due to being almost a straight line. This error could be fixed by performing more trials while compensating for the kerf.

Conclusion

It has been confirmed through experimental testing that, as expected, the resistance of a conductive material will increase at an exponential rate until it reaches infinite resistance as pores are cut into the sheet. It is shown that theoretical determinations of percolation thresholds match up well with what is seen experimentally. It is also concluded that there is an inverse relationship between the aspect ratio of an ellipse and the percolation threshold. Using this information obtained in this study, future work can be performed using different pore shapes or on different conductive materials to determine different transport properties of interest. We also show a new method developed to cut pores into a conductive sheet. This method allows percolation studies to be performed experimentally using pores of any shape.

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